Nonlinear Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Fuzzy set based models and methods of multicriteria group decision making

P. Ekel [a], J. Queiroz [a], R. Parreiras [a,*], R. Palhares [b]

[a] Pontifícia Universidade Católica de Minas Gerais, Av. Dom Jose Gaspar, 500, 30535-610, Belo Horizonte, MG, Brazil
[b] Universidade Federal de Minas Gerais, Av. Antonio Carlos, 6627, 31270-010, Belo Horizonte, MG, Brazil

ARTICLE INFO

Keywords:
Consensus schemes
Multiperson multicriteria decision making
Fuzzy preference relations

ABSTRACT

In the paper, the term *consensus scheme* is utilized to denote a dynamic and iterative process where the experts involved discuss a multicriteria decision problem. This discussion process is conducted by a human or artificial moderator, with the purpose of minimizing the discrepancy between the individual opinions.

During the process of decision making, each expert involved must provide preference information. The information format and the circumstances where it must be given play a critical role in the decision process. This paper analyses a generic consensus scheme, which considers many different preference input formats, several possible interventions of the moderator, as well as admitting several stop conditions for interrupting the discussion process. In addition, a new consensus scheme is proposed with the intention of eliminating some difficulties met when the traditional consensus schemes are utilized in real applications. It preserves the experts' integrity through the intervention of an external person, to supervise and mediate the conflicting situations. The human moderator is supposed to interfere in the discussion process by adjusting some parameters of the mathematical model or by inviting an expert to update his opinion. The usefulness of this consensus scheme is demonstrated by its use to solve a multicriteria group decision problem, generated applying the Balanced Scorecard methodology for enterprise strategy planning. In the illustrating problem, the experts are allowed to give their preferences in different input formats. But the information provided is made uniform on the basis of fuzzy preference relations through the use of adequate transformation functions, before being analyzed. The advantage of using fuzzy set theory for solving multiperson multicriteria decision problems lies in the fact that it can provide the flexibility needed to adequately deal with the uncertain factors intrinsic to such problems.

1. Introduction

Diverse types of uncertainty are often encountered in a group decision process. The main uncertain factors are related to the decision-makers' (DM) role, the preference for alternatives, and the judgments concerning the criteria involved [1]. There must be a rigorous correspondence between the uncertainty level of the acquired information on the group decision problem and the uncertainty level of its corresponding mathematical representation. Otherwise, the validity of the solutions obtained under the analysis of such models may be reduced. The use of fuzzy set theory for solving multiperson multicriteria decision problems may provide the flexibility needed to adequately deal with such uncertain factors [1–3].

* Corresponding author.
E-mail addresses: ekel@pucminas.br (P. Ekel), jqueiroz@pucminas.br (J. Queiroz), roberta.parreiras@terra.com.br (R. Parreiras), palhares@cpdee.ufmg.br (R. Palhares).

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doi:10.1016/j.na.2008.11.087

Group decision making is the process of achieving a solution based on the input and feedback of multiple experts. A group satisfactory solution is the one that is the most acceptable to the group as a whole, as the best solution almost never exists. Indeed, when a team of experts takes part in the decision process, it is quite natural that their opinions differ. Frequently, each member of the group has different information at hand and partially shares the goals of the other members. As a result, in order to increase the level of overall satisfaction for the solution across the group, all members must have their fair chance to influence the decision. Taking all of this into consideration, it would be a very simplistic approach to just aggregate the individual preferences into a collective preference, without first trying to minimize the divergence among opinions, by asking the more discordant experts to acquire further information on the problem and update their opinions.

In this paper, as in [1], we assume that consensus in decision making means that all members genuinely agree that the decision is acceptable. The term *consensus scheme* is utilized here to denote a dynamic and iterative process, where the experts involved discuss the problem. This discussion process is conducted by a human or artificial moderator, with the purpose of minimizing the discrepancy between the individual opinions. In general, at each cycle, the least concordant specialist is invited to acquire more information on the problem and review his opinion.

During the process of decision making, each DM involved must provide preference information. The information format and the circumstances where it must be given play a critical role in the decision process. This paper analyses a generic consensus scheme, which considers many different preference input formats, several possible interventions of the moderator, as well as admitting several stop conditions for interrupting the discussion process. Emphasis is given to fuzzy set based models and methods of multicriteria group decision making. Basically, the consensus schemes follow two traditional approaches (see, for instance [4–7]):

- The experts' opinions are combined into a collective opinion, using a weighted aggregation operator. Each weight (or importance coefficient) associated with each opinion is adjusted in such way that the discrepancy between the collective opinion and each individual opinion is minimized. In this approach, the most discordant expert is identified (and invited to update their opinion) only after these coefficients are adequately modified. The main disadvantages of this approach are: the opinion of a discordant expert, but with deep knowledge on the problem, can be neglected by the excessive reduction of the weight associated with his opinion; the process of obtaining an adequate set of weights may demand great computational effort.
- The weight associated with each opinion is kept fixed during the whole discussion process. The consensus is achieved only by asking the more discordant experts to update their opinions. This approach also has some negative aspects: in order to achieve an adequate level of consensus, a discordant expert may have to change drastically his initial position (maybe in an unjustified way); the experts may be repeatedly invited to review their respective opinions. In this way, this approach frequently demands further intellectual effort from the professionals involved.

In this context, a new consensus scheme is proposed with the intention of avoiding some undesirable situations which may happen in practice. An external person plays the role of a human moderator, who is supposed to interfere in the discussion process, whenever it is necessary, by adjusting some parameters (the importance coefficients associated with each expert and/or to each decision criterion) of the mathematical model being used or by inviting an expert to update his opinion. The helpfulness of this consensus scheme is demonstrated by its use for solving a multicriteria group decision problem, generated by applying the Balanced Scorecard methodology [8] for enterprise strategy planning. In the group decision problem considered, the experts are supposed to give their preferences in different input formats. But the information provided is made uniform on the basis of fuzzy preference relations, under adequate transformation functions, before being analyzed [4]. The major contribution of the present work lies in the fact that it contains guidelines that can be easily included into other existing consensus schemes.

2. Multiperson multicriteria decision problems

Multicriteria group decision making is the process of achieving a solution based on the input and feedback of multiple experts, taking into account several concurrent criteria. A typical multiperson multicriteria decision problem involves the following basic elements [7]:

- *The set of alternatives* $X = \{X_1, \ldots, X_n\}$. The set X corresponds to a finite and discrete list with more than one feasible alternative. The relevant characteristics (or attributes) of each alternative may be quantitative or qualitative.
- *The set of criteria* $C = \{C_1, \ldots, C_q\}$. The set C contains two or more criteria. Each criterion corresponds to a viewpoint, according to which the alternatives are evaluated and compared. They can be of quantitative or qualitative nature and can have more or less importance to the decision.
- *The set of decision makers* $E = \{E_1, \ldots, E_m\}$. The set E contains more than one expert. Usually each expert has their own perspective, motivation, and priority, which may result in conflicting preferences or opinions. Moreover, each expert plays a role in the group decision process, which is determined by their knowledge, experience, and intuition on the problem or authority in the enterprise.

Apart from those basic elements, the question raised by the problem must be specified. This work deals with decision methods suitable for solving ranking problems, where the alternatives must be ranked from best to worst, and choice problems, where the best alternative(s) must be selected.

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3. Consensus schemes

In an organization, the decisions often require multiple perspectives of different experts, as a unique person may not have enough knowledge on the problem to solve it alone. Since each expert often has different information at hand and a distinct subjective perception of the problem (which results in different opinions and preferences), the group should aim at achieving a satisfactory solution, rather than the best solution, as it hardly exists in practice. Classically, the term consensus is defined as a unanimous concordance among all individuals involved. But, in practice, such a definition is unsuitable for three main reasons [4]: it distinguishes only two states, the existence or nonexistence of consensus; the chances of achieving such a level of concordance are very low; in practical situations, it is not necessary to achieve such a level of concordance.

In view of that, a concept of *consensus level*, denoted as the *index of soft consensus*, was proposed [9]. Inspired by this key idea, several researchers have developed consensus schemes, which can be defined as dynamic and iterative discussion among the experts, with the purpose of minimizing the discrepancy between their respective opinions [4,10]. This process is coordinated by a moderator, who guides the discussion process, supported by indexes that reflect the level of consensus among the specialists and the level of concordance between each individual opinion and the collective opinion. Generally, such measures are computed on the basis of the experts' opinions or on the basis of the ranking of the alternatives obtained from these opinions. Some important aspects concerning consensus schemes are analyzed in further detail in the following subsections.

3.1. Different types and levels of influence of the professionals involved

Sometimes, it is relevant to consider the different levels of influence of each expert's opinion in determining the collective opinion. However, it is interesting to point out two types of influence. One is related to the expert's role in the group, which is determined by his knowledge and experience on the problem or authority in the enterprise. For instance, the opinion of a professional who has a deeper knowledge on the problem or who has more authority in the enterprise may be considered more relevant than the others. The most common way to include such influence in the consensus scheme is by assigning a different importance coefficient (weight) to each member of the group. These weights are usually specified by the moderator, before the beginning of the discussion process [1,4,7]. The other kind of influence is associated with the experts' opinions, being proportional to the level of concordance between each expert and the rest of the group. Such influence also depends on the way the aggregation operator deals with concordance measures: it may emphasize the more concordant opinions (which is the most common attitude) [11–13]; it may give equal emphasis to all opinions or may attach more importance to the most discordant opinions [13]. In [5,6], this second kind of influence is implemented through the use of scalar weights, which are especially chosen, in a systematic way, with the purpose of maximizing the level of consensus.

3.2. Input of the preference information

Each professional involved in the decision process has his own perception of the problem and usually has access to different information. As a consequence, it is quite natural to meet, in practice, circumstances where each DM feels more comfortable using a different preference format to express his preferences. Even the same DM may prefer to express his preference concerning each criterion in a different way. The input of the preference information can become a critical step in the consensus scheme if any expert is enforced to construct his preferences using a preference structure with which he does not feel comfortable. It is worth noting that each expert may have to construct preferences articulations more than once, if they are asked to review their opinion. The subjective and personal choice of an adequate format should be made on the basis of several aspects, such as easy assessment, intuitive appeal, and acceptable precision level. Reference [4] points out four preference structures that can be utilized: preference orderings, utility functions, fuzzy preference relations, and multiplicative preference relations. Reference [14] considers four additional possibilities: fuzzy estimates or linguistic terms, selected subset of X, fuzzy selected subset of X, and normal preference relation. When the consensus scheme admits such flexible and versatile information input, the information provided must be made uniform under adequate transformation functions, before being aggregated. For instance, the fuzzy preference relations and the multiplicative preference relations are chosen as the base elements for the uniform representation in [4,14], respectively.

Next, the most common preference structures are briefly described, given that the indexes $e \in \{1, 2, \ldots, m\}$ and $c \in \{1, 2, \ldots, q\}$ identify each expert from the set E and each criterion from the set C, respectively:

- *Preference orderings*. The preference on X is given as a preference ordering from best to worst. It can be represented as an ordered array $O_c^e = \{o_c^e(1), \ldots, o_c^e(n)\}$, where $o_c^e(.)$ is a permutation function over the integer values $\{1, 2, \ldots, n\}$.
- *Utility functions*. The preference on X is given as a set of n utility values $U_c^e = \{u_c^e(X_i), \ldots, u_c^e(X_n)\}$, where $u_c^e(X_i) \in [0, 1]$ represents the utility value assigned to alternative X_i.
- *Multiplicative preference relations*. The preferences are given as an $n \times n$ matrix M_c^e of positive preference relations $m_c^e(X_i, X_j)$ that reflects the preference intensity ratio between alternatives X_i and X_j, being understood as: X_i is $m_c^e(X_i, X_j)$ times as good as X_j. The preference intensity can be specified on the basis of several ratio scales. In the application example of this paper, the one suggested by Saaty [15] is utilized.
- *Fuzzy estimates*. The elements of X are evaluated with the use of fuzzy estimates $L_c^e = \{l_c^e(X_i), \ldots, l_c^e(X_n)\}$, where $l_c^e(X_i)$ is the fuzzy estimate associated with the alternative X_i. $l_c^e(X_i)$ can simply correspond to a fuzzy number or to a linguistic

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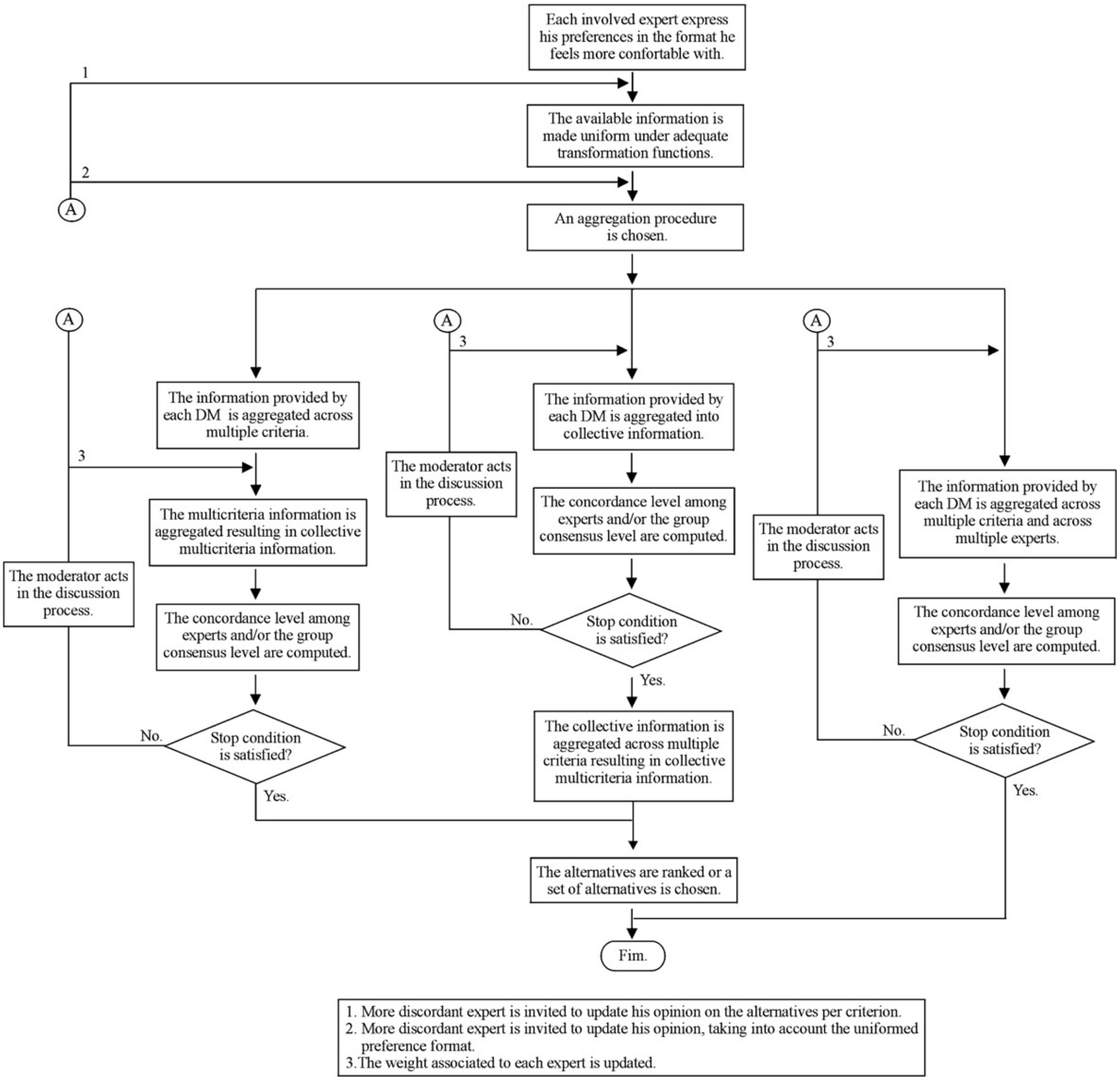

Fig. 1. Generic consensus scheme.

term from a set S [14]. For instance, in the application example from this paper, $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\}$, where s_0 = "incomparable", s_1 = "considerably worse", s_2 = "certainly worse", s_3 = "a little worse", s_4 = "worse", s_5 = "equivalent", s_6 = "better", s_7 = "a little better", s_8 = "certainly better", s_9 = "considerably better", s_{10} = "superior". Further discussion on the process for deriving meaningful membership functions can be found in [16].

- *Fuzzy preference relations*. The preference is described by a fuzzy preference relation $R_c^e \subset X \times X$. Its membership function $\mu_{R_c^e}(X_i, X_j) : X \times X \to [0, 1]$ indicates the degree to which the alternative X_i weakly dominates over (or is at least as good as) X_j. A rational (from substantial as well as psychological points of view) approach to constructing fuzzy preference relations is discussed in [17].

3.3. Multiperson and multicriteria aggregation modes

The aggregation across multiple criteria and the aggregation across multiple experts can be performed at different points in the course of the discussion process. Depending on the aggregation mode utilized, the consensus is computed on the basis of data of differing nature. Three possibilities are considered (they are highlighted in the diagram from Fig. 1):

- First, the aggregation is performed across all criteria, resulting in a global preference for each expert. Later, the information is aggregated across multiple experts. The consensus and concordance measures are based on multicriteria preference information. Reference [18] follows this approach.

- First, the aggregation is performed across multiple experts, in such a way that a collective opinion per criterion is obtained. Consequently, the consensus and concordance measures can focus on each criterion individually. After an acceptable consensus level is achieved, the collective information is aggregated across multiple criteria. See [7], for instance.
- The aggregation across multiple criteria and the aggregation across multiple experts are (practically) simultaneous. The consensus/concordance indexes are computed on the basis of each ranking obtained. This approach is addressed in [4,10,19], for instance.

The first and second approaches listed above can be grouped under the name "aggregation of individual judgments" [19]. They have in common the fact that they consider the group of experts as a unique individual, so they obtain collective judgments before generating the ranking of the alternatives. On the other hand, as the third approach generates a separate ranking based on the preferences of each expert (the group is considered as a collection of individuals), it can be denoted as "aggregation of individual rankings".

3.4. Moderator interventions

As already mentioned, the moderator intervention is based on consensus and concordance indexes, which can be computed at different moments during the whole group decision making process. As shown in Fig. 1, it can be done before or after the aggregation of the preferences across multiple criteria or after the alternatives are ranked per criterion. Besides, the moderator can interfere in the discussion process in different ways:

- The intervention can correspond to a request to the most discordant expert to update his opinion. This expert may give their preferences using the same preference format that he utilized at the beginning of the group decision making process (see the arrow 1 in Fig. 1) or using the uniformed preference format (see the arrow 2 in Fig. 1).
- The intervention can correspond to the execution of a procedure such as the ones from [5,6,13] to update the weights of each of the experts.

3.5. Stop conditions

The whole discussion process can be interrupted when any of the following conditions are fulfilled:

- An acceptable concordance (or consensus) level among the specialists is achieved.
- The previously specified maximum number of iterations is achieved.
- The same expert remains as the most discordant one after a specific number of subsequent iterations and the moderator cannot persuade that expert to change his opinion any further.
- In real time applications, after a substantial interval of time, the process must be interrupted, even though an acceptable concordance (or consensus) level has not been achieved yet.

4. A new consensus scheme

The major contribution of the proposed consensus scheme lies in the fact that it avoids two extreme undesirable situations that may happen in practice. It preserves each expert from being neglected or, else, excessively required. It is accomplished through the intervention of a human moderator to supervise and mediate the conflicting situations. As will be shown with an illustrative example, the new consensus scheme requires the execution of the following main steps:

Step (1) All experts are asked to provide their respective opinions.

Step (2) A collective opinion is obtained.

Step (3) A concordance measure is computed in order to rank the experts from the most discordant to the least discordant one. A consensus measure is computed in order to evaluate whether a satisfactory consensus level is achieved.

Step (4) If the consensus level is unsatisfactory, the most discordant expert is identified. However, if the same expert is identified as the most discordant one for successive iterations, the moderator is allowed to decide among the following actions:

- Invite the same expert again.
- Drive the invitation to the next most discordant expert.
- Re-evaluate the experts and/or criterion importance coefficients and update them.

It is interesting to emphasize some characteristics of the proposed consensus scheme:

- It contains guidelines that can be easily included into any existing consensus scheme that follows the same basic steps as are depicted in the diagram from Fig. 1.
- The importance coefficients associated with each expert and/or criterion can be conveniently updated during the discussion process by the moderator (not automatically by the software). Consequently, the consensus can be achieved through a more homogeneous discussion process, in which no expert has excessive demands placed on them, and no viewpoint is excessively valued or neglected.

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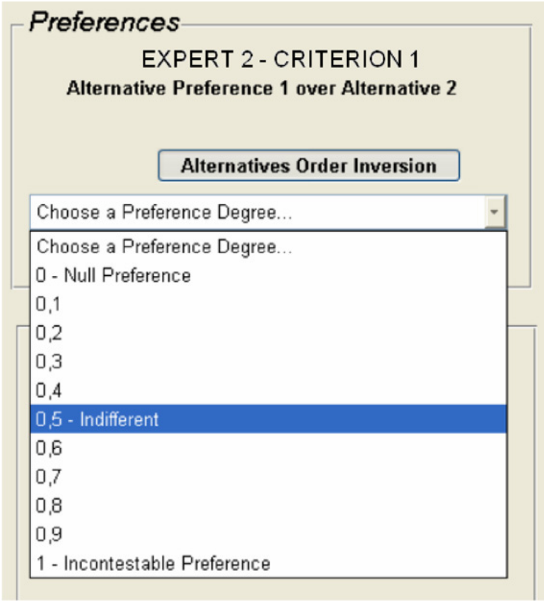

Fig. 2. Fuzzy preference relation interface.

- The moderator intervention can cause a computational effort reduction in situations where the opinion of the most important expert (i.e. the one with highest importance coefficient) is in disagreement with the majority opinion. In such situations the computational procedures for automatically changing the importance coefficients tend to demand relatively high computational effort.

The consensus schemes described in [4,6], as well as the one proposed here, were implemented as a computational system for solving multicriteria multiperson decision problems using MATLAB 7.0. The availability of many consensus schemes allows the moderator to utilize the one that he feels most comfortable with. The consensus scheme described in [4] has the following main characteristics: the most discordant expert at each cycle of the discussion process is identified and invited to review his opinion; an artificial moderator indicates how the discordant expert should modify his opinion (by enhancing or reducing their preference for a certain alternative); the importance coefficients associated with each expert are kept fixed during the whole discussion process. The consensus scheme described in [6], on the other hand, involves a computational moderator which automatically updates the importance coefficient of each expert, in order to achieve a satisfactory consensus level.

The system developed permits one to simulate different decision scenarios, as well as to validate and analyze the advantages and disadvantages of those consensus schemes. A hypothetical discussion among experts, supported by the software implemented, is described in the next subsection.

4.1. Application example

Consider a group decision problem where the enterprise's board of directors, which includes five members $\{E_1, E_2, \ldots, E_5\}$, is to plan the development of large projects (strategy initiatives) for the following five years. Four possible projects $\{X_1, X_2, X_3, X_4\}$ have been marked. It is necessary to compare these projects to select the most important of them, taking into account four criteria (categories) suggested by the Balanced Scorecard methodology [8] (it should be noted that all of them are of maximization character):

(C_1) Financial perspective.
(C_2) The customer satisfaction.
(C_3) Internal business process perspective.
(C_4) Learning and growth perspective.

Each expert is supposed to give their preferences on the basis of different preference structures. In view of that, the software developed has different dialog boxes conveniently designed to permit the input of preference information in terms of fuzzy estimates, linguistic terms, as well as the four preference formats considered in [4]. For instance, Fig. 2 shows the dialog box utilized to enter preferences in terms of fuzzy preference relations.

The experts' preferences are as follows:

- Expert E_1 (preference ordering):

$$O_1^1 = \{2, 1, 3, 4\}, \qquad O_2^1 = \{2, 3, 4, 1\}, \qquad O_3^1 = \{4, 2, 3, 1\}, \qquad O_4^1 = \{3, 4, 1, 2\}.$$

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- Expert E_2 (multiplicative preference relations):

$$M_1^2 = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 1/2 & 1 & 1/3 & 2 \\ 1/3 & 3 & 1 & 5 \\ 1/6 & 1/2 & 1/5 & 1 \end{bmatrix}, \qquad M_2^2 = \begin{bmatrix} 1 & 4 & 5 & 8 \\ 1/4 & 1 & 3 & 7 \\ 1/5 & 1/3 & 1 & 3 \\ 1/8 & 1/7 & 1/3 & 1 \end{bmatrix},$$

$$M_3^2 = \begin{bmatrix} 1 & 4 & 2 & 3 \\ 1/4 & 1 & 2 & 1/6 \\ 1/2 & 1/2 & 1 & 3 \\ 1/3 & 6 & 1/3 & 1 \end{bmatrix}, \qquad M_4^2 = \begin{bmatrix} 1 & 2 & 6 & 9 \\ 1/2 & 1 & 3 & 6 \\ 1/6 & 1/3 & 1 & 1/3 \\ 1/9 & 1/6 & 3 & 1 \end{bmatrix}.$$

- Expert E_3 (utility functions):

$$U_1^3 = \{0.6, 0.1, 0.9, 0.5\}, \qquad U_2^3 = \{0.4, 0.2, 0.8, 0.8\}, \qquad U_3^3 = \{0.1, 0.3, 0.3, 0.6\}, \qquad U_4^3 = \{0.8, 0.1, 0.5, 0.3\}.$$

- Expert E_4 (fuzzy preference relations):

$$R_1^4 = \begin{bmatrix} 0.5 & 0.7 & 0.9 & 0.8 \\ 0.3 & 0.5 & 0.8 & 0.8 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.8 & 0.5 \end{bmatrix}, \qquad R_2^4 = \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.1 \\ 0.6 & 0.5 & 0.8 & 0.7 \\ 0.7 & 0.2 & 0.5 & 0.4 \\ 0.9 & 0.3 & 0.6 & 0.5 \end{bmatrix},$$

$$R_3^4 = \begin{bmatrix} 0.5 & 0.4 & 0.2 & 0.2 \\ 0.6 & 0.5 & 0.9 & 0.8 \\ 0.8 & 0.1 & 0.5 & 0.3 \\ 0.8 & 0.2 & 0.7 & 0.5 \end{bmatrix}, \qquad R_4^4 = \begin{bmatrix} 0.5 & 0.6 & 0.4 & 0.6 \\ 0.4 & 0.5 & 0.4 & 0.7 \\ 0.6 & 0.6 & 0.5 & 0.6 \\ 0.4 & 0.3 & 0.4 & 0.5 \end{bmatrix}.$$

- Expert E_5 (fuzzy estimates, where the linguistic terms s_i, for $i \in \{0, 1, 2, \ldots 10\}$, were already defined in Section 3.2):

$$L_1^5 = \begin{bmatrix} s_5 & s_2 & s_6 & s_1 \\ s_8 & s_5 & s_1 & s_8 \\ s_4 & s_9 & s_5 & s_8 \\ s_9 & s_2 & s_2 & s_5 \end{bmatrix}, \qquad L_2^5 = \begin{bmatrix} s_5 & s_4 & s_7 & s_8 \\ s_6 & s_5 & s_2 & s_7 \\ s_3 & s_8 & s_5 & s_7 \\ s_2 & s_3 & s_3 & s_5 \end{bmatrix},$$

$$L_3^5 = \begin{bmatrix} s_5 & s_4 & s_4 & s_3 \\ s_6 & s_5 & s_6 & s_3 \\ s_6 & s_4 & s_5 & s_6 \\ s_7 & s_7 & s_4 & s_5 \end{bmatrix}, \qquad L_4^5 = \begin{bmatrix} s_5 & s_7 & s_6 & s_8 \\ s_3 & s_5 & s_7 & s_9 \\ s_4 & s_3 & s_5 & s_6 \\ s_2 & s_1 & s_4 & s_5 \end{bmatrix}.$$

All information provided by the experts on the basis of different preference structures is transformed into fuzzy preference relations, under adequate transformation functions [4]. Then, these are aggregated across multiple criteria. Here, the weighted sum

$$R^e = \sum_{i=1}^{q} wc_i R_i^e \tag{4.1}$$

is utilized.

In (4.1), wc_i is the importance coefficient (a scalar number satisfying $wc_i \in [0, 1]$ and $\sum_{i=1}^{q} wc_i = 1$) associated with the ith criterion; R_i^e is the matrix of fuzzy preference relations obtained from the preference judgments given by the Eth expert, taking into account the Cth criterion. In this example, the importance coefficients relating to each criterion are fixed by the moderator as $wc_1 = 0.4$, $wc_2 = 0.3$, $wc_3 = 0.2$, $wc_4 = 0.1$. The resulting matrices obtained on the basis of (4.1) for each expert are given by

$$R^1 = \begin{bmatrix} 0.5 & 0.43 & 0.6 & 0.47 \\ 0.57 & 0.5 & 0.67 & 0.53 \\ 0.4 & 0.33 & 0.5 & 0.37 \\ 0.53 & 0.47 & 0.63 & 0.5 \end{bmatrix}, \qquad R^2 = \begin{bmatrix} 0.5 & 0.74 & 0.78 & 0.90 \\ 0.26 & 0.5 & 0.53 & 0.65 \\ 0.22 & 0.47 & 0.5 & 0.75 \\ 0.09 & 0.34 & 0.25 & 0.5 \end{bmatrix},$$

$$R^3 = \begin{bmatrix} 0.5 & 0.75 & 0.27 & 0.39 \\ 0.25 & 0.5 & 0.13 & 0.08 \\ 0.72 & 0.87 & 0.5 & 0.57 \\ 0.61 & 0.92 & 0.43 & 0.5 \end{bmatrix}, \qquad R^4 = \begin{bmatrix} 0.5 & 0.54 & 0.53 & 0.45 \\ 0.46 & 0.5 & 0.78 & 0.76 \\ 0.47 & 0.22 & 0.5 & 0.32 \\ 0.55 & 0.24 & 0.68 & 0.5 \end{bmatrix},$$

$$R^5 = \begin{bmatrix} 0.5 & 0.35 & 0.59 & 0.42 \\ 0.65 & 0.5 & 0.29 & 0.68 \\ 0.41 & 0.71 & 0.5 & 0.71 \\ 0.58 & 0.32 & 0.29 & 0.5 \end{bmatrix}.$$

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Table 1
The rankings of the alternatives from best to worst.

Expert	Ranking
E_1	$X_2 \succ X_4 \succ X_1 \succ X_3$
E_2	$X_1 \succ X_2 \succ X_3 \succ X_4$
E_3	$X_3 \succ X_4 \succ X_1 \succ X_2$
E_4	$X_2 \succ X_1 \succ X_4 \succ X_3$
E_5	$X_3 \succ X_2 \succ X_1 \succ X_4$
Group	$X_1 \succ X_3 \succ X_2 \succ X_4$

Table 2
Consensus levels and contribution indexes.

Consensus per alternative	$G_1 = 0.53, G_2 = 0.53, G_3 = 0.53, G_4 = 0.67$
Group consensus ($p = 1$)	$G_G = G_1 = 53.3\%$
Expert contributions	$D_1 = -0.23, D_2 = 0.27, D_3 = -0.73, D_4 = -0.07, D_5 = -0.57$

The weighted sum aggregation operator is also utilized to aggregate across multiple experts and obtain a collective matrix R^G of fuzzy preference relations. Initially, as the moderator considers all DM equally important ($we_i = 0.2, i = 1, \dots, 5$), R^G is given by

$$R^G = \sum_{i=1}^{m} we_i R^i = \begin{bmatrix} 0.5 & 0.56 & 0.55 & 0.53 \\ 0.44 & 0.5 & 0.48 & 0.54 \\ 0.44 & 0.52 & 0.5 & 0.54 \\ 0.47 & 0.46 & 0.46 & 0.5 \end{bmatrix}.$$

The q alternatives are ranked according to the degree of dominance, defined by

$$\mu^D (X_k) = \sum_{i=1}^{n} R_{ki}^G. \tag{4.2}$$

A higher value of $\mu^D(X_k)$ corresponds to a higher level of dominance (preference) of the kth alternative over the others. Table 1 shows the ranking of the alternatives obtained by applying (4.2) to the information provided by each expert separately and on the basis of the collective information.

The consensus level per alternative G_i and the consensus level G_G achieved by the group of experts are respectively given by Eqs. (4.3) and (4.4), as presented in [6]:

$$G_i = \sum_{j=1}^{m} \left[\left(1 - \frac{|O(X_i) - OE_j(X_i)|}{(n-1)} \right) we_j \right], \tag{4.3}$$

$$G_G = \frac{1}{p} \sum_{i=1}^{p} G_{[i]}. \tag{4.4}$$

In (4.3), we_j is the weight associated with the jth expert, $O(X_i)$ is a function that returns the position of alternative X_i, for $i \in \{1, \dots, n\}$, in the ranking obtained on the basis of the collective preferences; $OE_j(X_i)$ is a function that returns the position of alternative X_i in the ranking obtained on the basis of the preferences of the jth expert. In (4.4), $G_{[i]}$ represents the consensus level for the alternative ranked in the ith position. Thus, the group consensus level is computed considering the top p alternatives.

Here, the most discordant expert is identified through an index D_j that reflects the contribution of the jth expert to the collective evaluation of each alternative [6]:

$$D_j = \sum_{i=1}^{n} G_i - G_{i \setminus j}, \tag{4.5}$$

where $G_{i\setminus j}$ is the group consensus on alternative X_i without considering the jth expert, as given by Eq. (4.6), where $\beta_j = we_j / \sum_{i=1 \wedge i \neq j}^{m} we_j$. It is interesting to note that a higher D_j corresponds to a higher contribution of the jth expert.

$$G_{i\setminus j} = \sum_{j=1}^{m} \left[\left(1 - \frac{|O(X_i) - OE_j(X_i)|}{(n-1)} \right) \beta_j \right]. \tag{4.6}$$

In this example, a consensus level equal to or higher than 85% is considered satisfactory. But, as Table 2 shows, the group consensus level (which is calculated considering only the first ranked alternative) is lower than the minimum acceptable value.

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Table 3
Consensus levels and contribution indexes.

Consensus per alternative	$G_1 = 0.60, G_2 = 0.53, G_3 = 0.53, G_4 = 0.73$
Group consensus ($p = 1$)	$G_G = G_1 = 60\%$
Expert contributions	$D_1 = -0, 27, D_2 = -0, 10, D_3 = -0, 60, D_4 = -0, 10, D_5 = -0, 60$

Table 4
Consensus levels and contribution indexes.

Consensus per alternative	$G_1 = 0.67, G_2 = 0.60, G_3 = 0.53, G_4 = 0.73$
Group consensus ($p = 1$)	$G_G = G_1 = 66.7\%$
Expert contributions	$D_1 = -0.30, D_2 = 0.20, D_3 = -0, 30, D_4 = -0.13, D_5 = -0.47$

Table 5
Consensus levels and contribution indexes.

Consensus per alternative	$G_1 = 0.80, G_2 = 0.67, G_3 = 0.60, G_4 = 0.73$
Group consensus ($p = 1$)	$G_G = G_1 = 80\%$
Expert contributions	$D_1 = -0, 03, D_2 = -0, 03, D_3 = -0, 20, D_4 = -0, 20, D_5 = 0, 13$

The most discordant opinion is the one given by the expert E_3 (the one associated with the lower expert contribution). So, expert E_3 is invited to review his position. That expert's new preferences and the new collective preferences, as well as the resulting rankings of the alternatives, are given by

$$R_3 = \begin{bmatrix} 0.5 & 0.75 & 0.35 & 0.55 \\ 0.25 & 0.5 & 0.13 & 0.27 \\ 0.65 & 0.87 & 0.5 & 0.66 \\ 0.45 & 0.73 & 0.34 & 0.5 \end{bmatrix} \qquad R^G = \begin{bmatrix} 0.5 & 0.56 & 0.57 & 0.56 \\ 0.44 & 0.5 & 0.48 & 0.58 \\ 0.43 & 0.52 & 0.5 & 0.56 \\ 0.44 & 0.42 & 0.44 & 0.5 \end{bmatrix}$$

$$E_3 : X_3 \succ X_1 \succ X_4 \succ X_2 \qquad Group : X_1 \succ X_3 \succ X_2 \succ X_4.$$

Table 3 shows the updated consensus levels and contribution indexes. As there are two potential candidates for being invited, E_3 and E_5, the proposed scheme automatically invites expert E_5, saving expert E_3 the task of reviewing his opinion. The new preferences and the resulting rankings are as follows:

$$R^5 = \begin{bmatrix} 0.5 & 0.59 & 0.59 & 0.42 \\ 0.41 & 0.5 & 0.29 & 0.68 \\ 0.41 & 0.71 & 0.5 & 0.71 \\ 0.58 & 0.32 & 0.29 & 0.5 \end{bmatrix} \qquad R^G = \begin{bmatrix} 0.5 & 0.61 & 0.57 & 0.56 \\ 0.39 & 0.5 & 0.48 & 0.58 \\ 0.43 & 0.52 & 0.5 & 0.56 \\ 0.44 & 0.42 & 0.44 & 0.5 \end{bmatrix}$$

$$E_5 : X_3 \succ X_1 \succ X_2 \succ X_4 \qquad Group : X_1 \succ X_3 \succ X_2 \succ X_4.$$

According to the updated contribution values from Table 4, expert E_5 should be invited again as, currently, D_5 has the lowest value among all the expert contribution values. As the dialog box in Fig. 3 shows, the implemented consensus scheme allows the moderator to interfere in the process by choosing to invite the same expert to review his opinion again, to invite another expert or to change the importance coefficients. Assuming that the choice of the moderator is the second option, the expert E_1 is asked to review his opinion. The updated preferences and the resulting rankings are given by

$$R^1 = \begin{bmatrix} 0.5 & 0.63 & 0.6 & 0.47 \\ 0.37 & 0.5 & 0.47 & 0.53 \\ 0.4 & 0.53 & 0.5 & 0.37 \\ 0.47 & 0.6 & 0.57 & 0.5 \end{bmatrix} \qquad R^G = \begin{bmatrix} 0.5 & 0.65 & 0.57 & 0.57 \\ 0.35 & 0.5 & 0.44 & 0.55 \\ 0.43 & 0.56 & 0.5 & 0.57 \\ 0.43 & 0.45 & 0.42 & 0.5 \end{bmatrix}$$

$$E_1 : X_1 \succ X_4 \succ X_3 \succ X_2 \qquad Group : X_1 \succ X_3 \succ X_2 \succ X_4.$$

As Table 5 shows, the group consensus is still unsatisfactory.

Expert 4 is automatically invited to update his opinion (see Table 6):

$$R^4 = \begin{bmatrix} 0.5 & 0.54 & 0.53 & 0.45 \\ 0.46 & 0.5 & 0.78 & 0.76 \\ 0.47 & 0.22 & 0.5 & 0.6 \\ 0.55 & 0.24 & 0.4 & 0.5 \end{bmatrix} \qquad R^G = \begin{bmatrix} 0.5 & 0.65 & 0.57 & 0.56 \\ 0.35 & 0.5 & 0.44 & 0.58 \\ 0.43 & 0.56 & 0.5 & 0.62 \\ 0.43 & 0.45 & 0.37 & 0.5 \end{bmatrix}$$

$$E_4 : X_2 \succ X_1 \succ X_3 \succ X_4 \qquad Group : X_1 \succ X_3 \succ X_2 \succ X_4.$$

As expert E_3 had already changed his opinion, the moderator decides to redefine the experts' importance coefficients as $we_1 = 0.25$, $we_2 = 0.3$, $we_3 = 0.15$, $we_4 = 0.15$, $we_5 = 0.15$, which results in an acceptable consensus level, as Table 7 shows. As $p = 1$, when the best alternative achieves a satisfactory group consensus level, the discussion process is interrupted. The other alternatives do not necessarily have to achieve a satisfactory consensus level.

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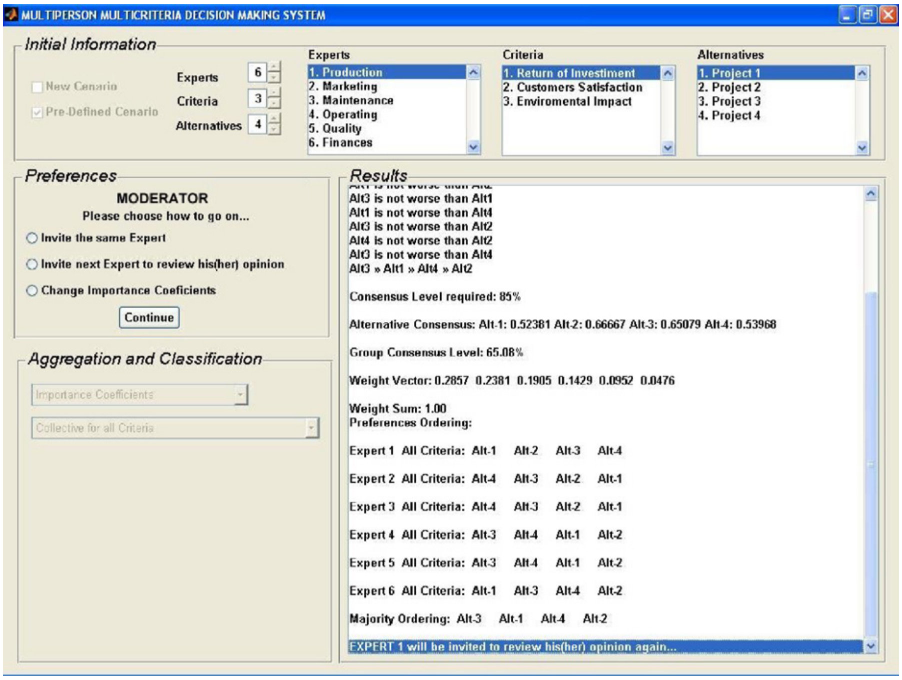

Fig. 3. System general view when the moderator is solicited.

Table 6
Consensus levels and contribution indexes.

Consensus per alternative	$G_1 = 0.80, G_2 = 0.73, G_3 = 0.60, G_4 = 0.80$
Group consensus ($p = 1$)	$G_G = G_1 = 80\%$
Expert contributions	$D_1 = -0.07, D_2 = 0.10, D_3 = -0.23, D_4 = 0.10, D_5 = 0.10$

Table 7
Consensus levels and final ranking.

Consensus per alternative	$G_1 = 0.85, G_2 = 0.75, G_3 = 0.58, G_4 = 0.78$
Group consensus ($p = 1$)	$G_G = G_1 = 85\%$
Final ranking	$X_1 \succ X_3 \succ X_2 \succ X_4$

It must be emphasized that, in this example, the moderator is supposed to be capable of setting such coefficients in a justified and rational manner. Such a hypothesis is essential to guarantee that the consensus index is capable of reflecting the quality of the achieved consensus. These coefficients must reflect the importance of the opinion of each expert for the construction of the collective opinion; otherwise, a low quality consensus (but with an acceptable value of the consensus index) may be constructed.

Consensus schemes are sensitive to the experts' importance coefficients. When the opinion of the experts with high importance coefficients differs from the opinion of the rest of the group, a high number of iterations may be required to achieve a satisfactory consensus level. But the sensitivity to such coefficients depends on the aggregation and ranking procedures utilized to generate the results.

When the initial conditions of the decision problem are changed during the discussion process, all experts must be invited to analyze the new scenario and give their opinions again. In other words, the discussion process must be restarted.

5. Conclusions

This paper addressed the use of a consensus scheme as a systematic (logical and sequential) instrument for regulating the group decision process as well as controlling the conflicts originated from the confrontation of different experts and achieving a consistent aggregation of their opinions. Several possible ways of implementing consensus schemes were analyzed and a new consensus scheme was proposed with the main intention of overcoming undesirable situations encountered in practical applications, when the traditional consensus schemes are being used. Such undesirable situations were illustrated by simulating a discussion process supported by an integrated decision support system developed in the MATLAB 7.0 environment. The group multicriteria decision problem considered was generated with the use of the Balanced

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Scorecard methodology for enterprise strategy planning. The simulated discussion process demonstrates the usefulness of the proposed consensus scheme.

Acknowledgements

This research was supported by the National Council for Scientific and Technological Development of Brazil (CNPq), grant 302406/2005-0, and the State of Minas Gerais Research Foundation (FAPEMIG), grant TEC 00140/07.

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